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DEPARTMENT OF OPERATIONAL RESEARCH

A COMPARATIVE STUDY OF METHODS FOR ESTIMATING OUTSTANDING CLAIMS IN GENERAL INSURANCE

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1. INTRODUCTION

1.1 Project Background

Claims in general insurance are subject to delay from the date that they are generated, to the ultimate date of settlement.

This happens because of delayed notification and/or delayed settlement from the date of such notification.

During this time, i.e. between the generation of a claim and its ultimate settlement, cost can be affected by a number of elements.

Elements, such as inflation, changes in court awards, changes in legislation are all outside the company's control.

These are coupled with changes in portfolios, claim-handling system etc., which happen within the company and can be controlled by it to a certain extent. Claims of the same type are also subject to a considerable random element.

At the end of each year, and perhaps more often, it is necessary to estimate the likely cost of all outstanding claims of any or all accounts.

Under the heading "outstanding" we include two types of claim: the first type includes those claims already reported to the company which have not been settled, the second type includes the claims generated in the past but not yet reported to the company. This latter type is also known as I.B.N.R. claims (Incurred But Not Reported).

The company has traditionally estimated the cost of each notified claim on an individual basis, using the experience and expertise of the claim's department. The I.B.N.R. Claims, together with the corresponding payments have been estimated separately.

The ability of using these estimates, for management purposes, is limited as the implicit margins cannot be easily evaluated for a given inflation assumption.

It is further impossible to evaluate the effect on these estimate *c* different inflation assumptions. Another disadvantage of the subject!· estimates for notified claims is that they are always conservative.

1.2 General Explanation of the Project

It is only the last few years that an objective technical approach to the problem of estimating outstanding claims has been developed.

Many methods have been devised, mainly by actuaries attempting to deal with the problem.

This project is about applying the three most popular of these methods to data of the Provincial Insurance Company and explore their relative merits.

2. DETAILED DESCRIPTION OF THE BACKGROUND AND OBJECTIVES OF THE PROJECT

2.1 The Risk Groups

For reporting purposes, the non life business of the company has been divided into sixteen fairly broad categories. These categories will be known hereafter as "risk groups". For the purpose of this project we had access to data for the following fourteen groups:

1. Liability - General (Public)
2. Liability - Employers
3. Goods in Transit
4. Motor: Private cars
5. Motor: Commercial Vehicles
6. Motor: Two wheeled vehicles
7. Motor: Trade risks
8. Motor: Fleets
9. Pecuniary loss: Fidelity Guarantee
10. Pecuniary loss: Miscellaneous
11. Personal accident
12. Property: Theft
13. Property: Miscellaneous
14. Yacht and Motor Boats.

2.2 ^Tabulating Claim Payments in Cohort Form _

The Insurance Companies act of 1974 requires statistical returns of claim payments for each risk group, in cohort form. A definition of cohort is: "A series of payments, over time, for claims generated (i.e. an accident happened) in the same year of origin and belonging to the same risk group. Each payment so tabulated can either be the ultimate settlement payment for a claim or a partial payment towards it.

We say that a payment is made in the i development year, if it is paid i years after the year that the claim was generated i.e. the accident happened. The year of origin is development year 0 (zero).

It must be made clear that if the insurance company recovers money with respect to a claim, the recovered sum counts as a negative payment.

2.3 The Runoff Triangle

Almost all the methods devised for dealing with the problem of estimating provision for outstanding claims, have been designed, in such a. way as to apply to data in the form of figure *I*,

•Figure 1. The Runoff Triangle

Year of development

		0	1	2	---------------	k
		C_{00}	C_{01}	C_{02}		C_{0K}
Year of Origin	1	c_{10}	c_{n}	c_{12}	---------	X
	„			'χ		
	K	C_{K0} /				

Where the c..'s represent the total actual payment made during development year j for claims generated in year of origin i.

The following points must be made clear:

(a) on each diagonal of the triangle the payments are made during the same office year.

(b) The number of development years cannot exceed the number of years of origin.

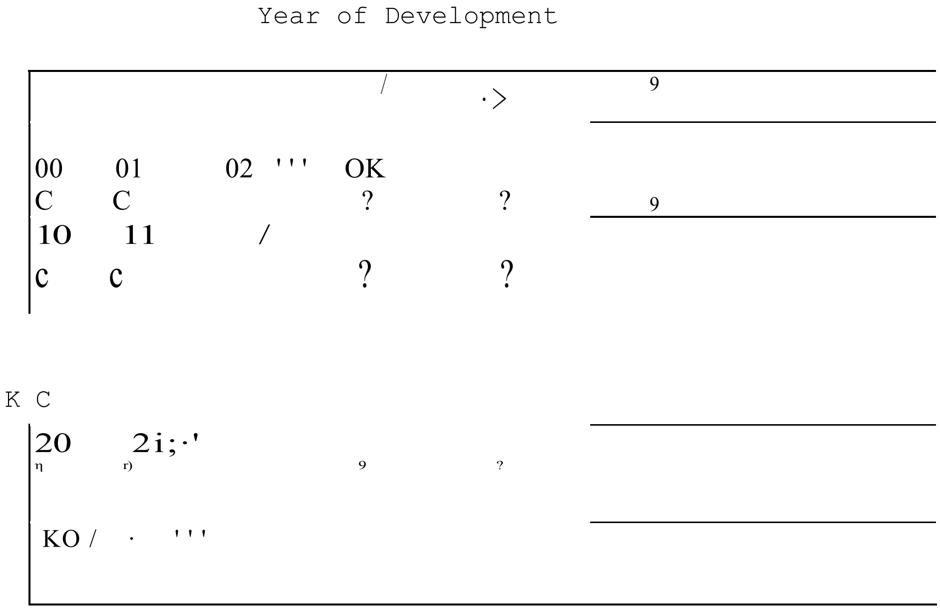
The methods for estimating provision for outstanding claims can easily be modified in order to deal with data presented in a "chopped triangle" form. This is the case when the number of development years is less than the number of years of origin, in such a way that the top right-hand corner of the triangle does not exist. The easiest way to overcome the difficulty is then to fill all empty cells with zeros.

2,4 Formulating the Estimation Problem

Having introduced to the reader the ideas of cohort and runoff triangle, we think that we now can formulate the estimation problem.

Consider a runoff triangle of actual payments with k years of origin. For the i year of origin the recorded data consist of payments made up to the end of development year (k-i). Outstanding payments for this cohort are to be made in development years (k-i+1), (k-i+2),..., Estimating provision for outstanding claims means to estimate these future payment^ for each cohort. Figure 2 can help the reader to visualize the estimation problem. The actual

Figure 2. Graphical Representation of the Estimation Problem



data is given to us by the C_{ij}'s in the runoff triangle. What we do not know is the future payments represented here by question marks. So to estimate provision for outstanding claims means to fill in the semi-infinite rectangle of figure 2. The rectangle is said to be semi infininte because, in theory, payments with respect to a particular year of origin can be made anywhere in the future. So, theoretically we have an infinite number of development years.

To formulate the problem, we need the following notation: Regard c_{ij}, the actual payment in the (i,j) cell of the run-off triangle as a realisation of the Random Variable C_{ij}, and let $\mu_{ij} = E(C_{ij})$.

PROBLEM; Use the data of the runoff triangle and any other relevant information to estimate the quantity

$$P = \sum_{i=0}^{k} \sum_{j=k-i+1}^{\infty} \mu_{ij}\, f(i+j-k), \text{ where } f(i+j-k)$$

are appropriate discounting factors allowing for inflation, investment income etc.. Throughout the project we did not allow for investment income, but only for inflation.

Although the problem is about estimating a single amount P, we can only estimate this reliably if we can identify a model giving μ_{ij}'s which fit the actual observed C_{ij}'s reasonably well. We therefore need to model the runoff triangle of observed payments. In all attempts to model the runoff triangle, the following two assumptions are made either explicitly or implicitly.

Assumption 1

In a stable environment, the runoff of each and every cohort develops in the same pattern, apart from random disturbance. The runoff of each cohort can thus be described by a set of factors.

$$p_{iJ} = \mu_{iJ} / \sum_{k=0}^{\infty} \mu_{ik}$$

independent of the year of origin i. The above factors are the expected payments in development year j as a proportion of the total payment for the whole cohort.

Assumption 2

In an unstable environment the runoff is the same as in a stable environment except that each payment is altered by an appropriate factor depending on the office year of payment.

The runoff of each cohort can now be described by a set of factors:

$$\frac{\mu_{iJ}}{\sum_{k=0} \mu_{iJ}} = \frac{\rho_J \lambda_{i+J}}{\sum_{k=0} \rho_J \lambda_{i+J}}$$

where X's are a set of exogenous influence factors. It is common to think of the X's as the values of an appropriate inflation index. The above factors are the expected payments in development year j as a proportion of the total payment for the whole cohort.

In practice the problem of estimating provision for outstanding claims has been divided into two sub-problems which have been treated fairly distinctly

(a) Estimating outstanding claims for development years 0 to k.

(b) Estimating outstanding claims for development years k+1 onwards.

This project has been concerned about problem (a), we had neither sufficient data nor time to deal with problem (b).

Going back to Figure 2, we tried to fill in those question marks corresponding to development years 0 to k.

2.5 Statistical Methods for Modelling the Observed Runoff Triangle and Estimating Provision for Outstanding Claims

In the previous chapter we stated that in order to calculate reliably provision for outstanding claims, one needs to model reasonably well the observec runoff triangle.

The statistical methods compared in this project have been tested with respect of how well they model the observed runoff triangle as well as in terms of estimates for outstanding claim payments. These methods are:

(a) The chain ladder method

(b) The cost per claim method

(c) The separation method.

The methods will be described briefly in this section. Mathematical formulae for estimates for outstanding claims, as well as formulae for the modelling of the observed runoff triangle by each method, will be given in Appendix A.

fa") The Chain Ladder Method (See also Appendix Al)

The basic feature of this method is that it ignores inflation. It was however modified slightly in order to allow for inflation. The modified method will be known hereafter as chain ladder method with inflation.

The assumption behind the chain ladder method is that payments for claims have a stable distribution over time, the same for every cohort. This distribution is deduced from past cohorts and appropriate proportional factors are derived for each year of development. These are then applied to payments up to date in order to model the observed runoff triangle as well as to estimate provision for outstanding claims.

The fact that inflation is ignored leads to underestimation and over-estimation in periods of high and low inflation respectively.

The chain ladder method with inflation follows exactly the same procedure, but first existing data are discounted to common currency values by using an appropriate inflation index. Derived values from applying the chain ladder method to the discounted data are then reflated back using the same inflation index and assuming suitable rates for future inflation.

(b) The cost per claim method (See also Appendix A2) In order for this method to be applied for modelling the observed runoff triangle and estimating provision for outstanding claims, it needs not only payments data but also number of open claims data i.e. it needs two runoff triangles, one for payments and one for open claims numbers.

The assumption behind the method is that not only payments but also numbers of open claims follow stable distributions, the same for each cohort.

The feature of this method is that it deduces separately the distribution of the number of open claims for each cohort. This is done by using the chain ladder method.

For each cohort and each development year, the payments are divided by the corresponding number of open claims and so a series of average cost per claim figures is derived. These, together with the deduced distribution of the number of open claims, are used for modelling the runoff triangle and estimating outstanding payments.

The method can allow for inflation in the same way as the chain ladder method with inflation i.e. by discounting average cost per claim figures to common currency values using appropriate inflation index and then reflating derived figures by the method using the same index and assuming suitable rates of future inflation.

(c) The Separation Method (See also Appendix A3)

The separation method assumes that payments follow a stable distribution over time.

The feature of the method is that it uses past data not only to deduce the proportional factors of the distribution of payments, but also to estimate past inflation. However future inflation is once again assumed subjectively.

2.6 Objectives of the Project^

It is already said that the objective of the project is to investigate the relative merits of the methods by applying them to data of Provincial Insurance.

The methods were to be tested with respect to the following:

(1) Fit of the model for the runoff triangle assumed by each method to the actual data.

(2) Reliability of forecasts of future payments given by each method.

(3) Robustness with respect to different forecasts for future inflation

(4) Reaction of forecasts given by each method to additional observatio

The project was not concerned about the theoretical properties of the methods. These have been presented in various papers and are well investigated by actuaries. Our sole purpose in this project was to advise Provincial about practical aspects of the performance of each method, in order for management to be in a better position with respect to deciding which of the methods to use.

The other objective of the project was to choose appropriate inflation indices for each risk group. In order to do so an extensive research was carried out. We had to understand fully the risk elements in each group in order to be able to choose appropriate inflation indices.

Of course other problems had to be attacked for the project to be successful. We had to program the methods in the University Computer, to choose appropriate statistical measures in order to compare the performance of the methods etc. These will be described in subsequent sections.

3. DATA COLLECTION AND HANDLING

3.1 Data Sources

Our main data sources were:

(a) Provincial's returns to the Department of Trade were used to collect data about past claim payments and numbers.

(b) Government publications were used in order to find inflation indices.

Provincial's returns were divided into returns for reported claims and returns for reopened claims. Under the heading reported, all claims notified to the company are included. The idea of reopened claims is rather different w Consider a settled claim. It may be the case that in a later stage either a sum has to be paid or to be recovered with respect to this claim. From the date that this is known to the date of resettlement, the claim is known as reopened. Reopened claims are treated as new claims in the returns for reported claims.

We were able to borrow for one week, folios containing the returns for both reported and reopened claims. Appendix B contains a sheet showing a typical example of the format of the returns. At the end of each year, known as reporting year, starting from 1970, and for each risk group a sheet of information similar to the one in Appendix B is produced. The returns for reopened claims are of similar format and start from 1972. We also had access to IBNR estimates for both payments and number of claims starting from 1970.

The government's publications used in order to find suitable inflation indices were found in the University Library. The "Monthly Digest of Statistics", "Economic Trends", were found as the most useful of these publications.

3.2 Data Capture

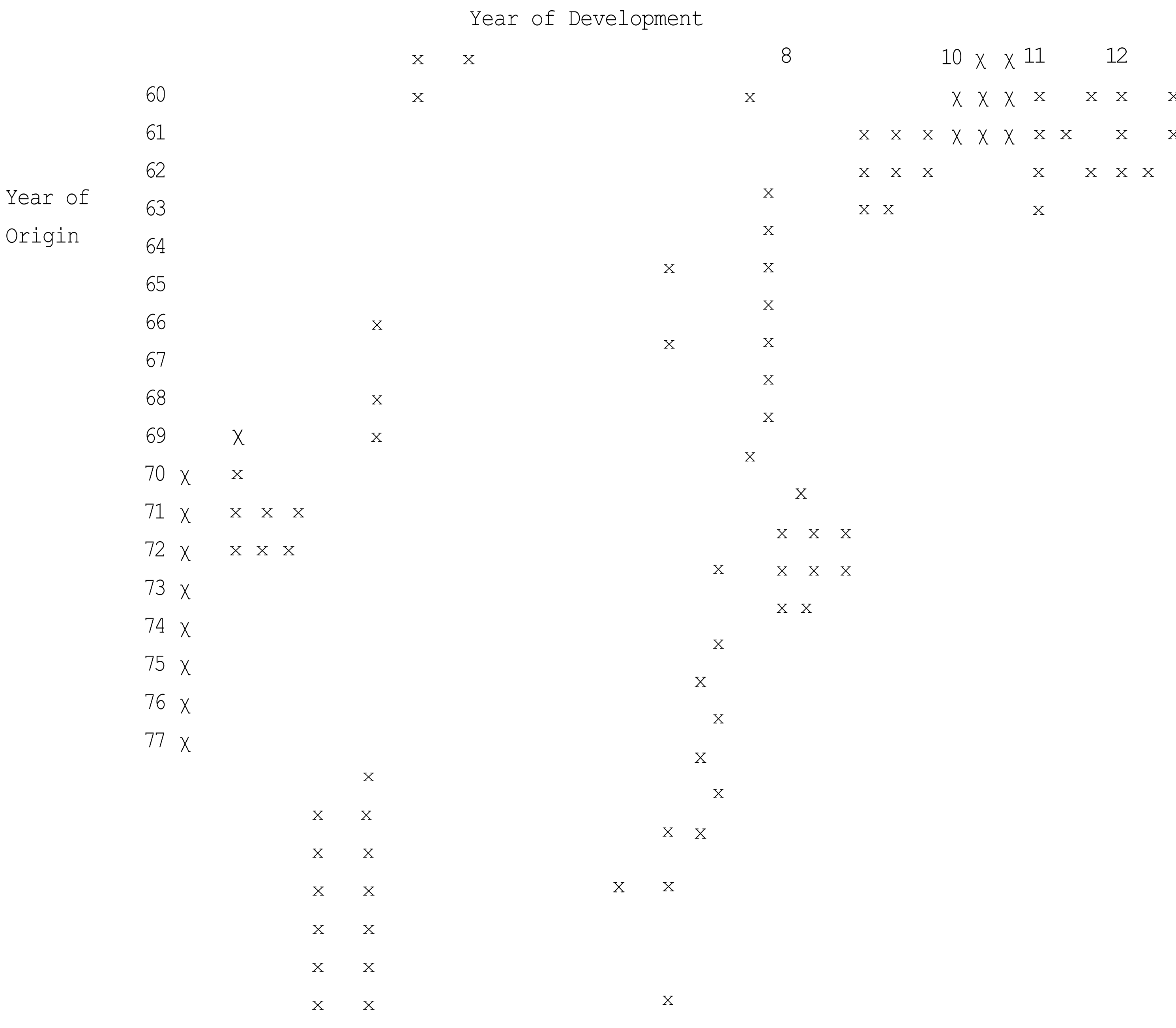
Unfortunately the only way that data from Provincial's returns could be transferred to the University's computing facilities was by encoding them on to special coding forms and punching them onto cards.

For each cohort on each reporting year the following information has been encoded from the returns for reported claims, reporting year
risk group | to specify the cohort
year of origin of claim,,

3. 3 Arranging Data into the Appropriate Form for the Application of the methods

Two files, one for reported claims and one for reopened claims, were opened in the Manchester University's Compute^ containing all our data from Provincial's returns. A computer programme was developed in FORTRAN IV in order to tabulate the data in suitable form for applying the methods. For each risk group, total payments, No. of settled claims and average cost per claim ([illegible] / No. of settled claims) were tabulated in the form shown in Figure 3. Reference to this program and complete documentation can be found in A.Kenney's dissertation.

FIGURE 3. FORM IN WHICH DATA IS TABULATED



16 17 χ

x x

The x's in figure 3 represent observed values.. For each risk group we have 3 such tabulations. One for total payments, one for No. of claims, one for average costs per claim.

The tabulation is divided by the horizontal line into two parts. The lower part is the runoff triangle used by all methods and the upper part to be used by the costs per claim method.

In order to apply the separation method, an estimate of the total number of claims for each cohort is necessary. The program could give this estimate in two ways:

(a) For each cohort the notified claims at the end of the development year zero were added to the IBNR estimates for the number of claims made at the end of this year.

(b) An alternative estimate was that for each cohort, notified claims during development years corresponding to office years, up to 1977 were added to IBNR estimates for the number of claims made at the end of 1977, after allowance had been made for reopened claims not to be counted twice. These estimates were only possible from 1972, as this was the first year that returns for reopened claims existed.

The two alternative estimates were so close as to be of no consequence at all which one we used. We preferred the first estimate, which we named "first year reported plus IBNR" because it was available for all cohorts 1970-77 and after consulting John Taylor.

This was the only case in which data for reopened claims was involved in the project.

3.4 First Look at the Data - Comments

Towards the end of 1977 a change in the definitions of settled claims took place. Up to that time a claim was settled when all anticipated payments had been made. The new definition did not define a claim as settled unless, not only payments but also anticipated recoveries had been made. This had the following effects:

(i) A lower proportion of claims will be settled than previously, (ii) Ultimate claims cost will reflect the recoveries previously excluded (iii) At the end of 1977, a substantial number of claims subject to recovery were reopened. This inflated the number of reopened claims for 1977.

Having the above made clear, the next step was to discount our average cost per claim figures back to 1970 values. To do so we used the Retail Prices Index for risk groups that included prices and Earnings Indices where Earnings were included. When there no obvious choice both R.P.I, and Earnings Indices were used.

By discounting all figures into common currency values we were able to point out peculiar movements, trends and other characteristics we thought interesting. The objective was to better understand the business in each group.

The above points were discussed with John Taylor and he suggested an error in our definition of average cost per claim. He suggested, as most appropriate, the definition

$$\text{"average cost per claim"} = \frac{\text{Total payments}}{\text{No. of open claims}}$$

This definition treats all claims as equal irrespective of whether they were paid for partly or totally. He further answered all our questions and he arranged meetings with Mr. D. Alexander and Mr. French of the accident and transit department respectively, for further discussion of several points.

4. APPLICATION OF METHODS AND COMPARISON OF RESULTS

4.1 Towards application of methods

The initial program was modified in order to accommodate the new definition for the average cost per claim values. This new definition had the effect of deflating the average cost per claim figures as the number of claims in the denominator was greater than or equal to the number of claims under the old definition. Fortunately no effect was observed on the trends of average cost per claim figures caused by the introduction of the new definition. Hence the points raised in the discussions with John Taylor, Mr. Alexander and Mr. French were still valid.

In order to-apply the methods to data and compare results, the following steps had to be taken:

(a) to program the methods in FORTRAN IV;

(b) to undertake a considerable amount of economic research with the scope of choosing appropriate inflation indices for the risk groups and, having done this, to forecast further inflation. My responsibility was the Economic Research.

4.2 Economic Research

In General Insurance, all risks refer to material goods subject to loss or damage on the one hand and to lost earnings on the other. For the first category, price inflation indices are appropriate. For the second, Earnings indices are appropriate. The prices indices considered in this project were: Retail Prices Index (R.P.I.) and the Wholesale Prices Index (W.P.I.). The Earnings Indices considered were: The Average Earnings Index (A.E.I.) and the Index of Basic weekly Earnings for manual workers (M.W.I.) initially. In a later stage a third earnings index was considered, namely Index of Wages and Salaries per unit of output (I.W.P.U.O.).

Under the basic assumption that payments were distributed, in terms of money value, uniformly over time and that each and every year had twelve equal months, the inflation figure for a particular year was the average of the twelve monthly figures. Forecasts were made under the same assumption.

On the basis cf the above assumption we were able to obtain inflation figures as follows: RPI: for years 1948-1977, source Economic Trends. WPI: for years 1954 - 1977, source Economic Trends, AEI: for years 1963-1977, source, Dept. of Employment Gazette. M.W.I.: for years 1956-1977, source, Dept. of Employment Gazette. I.W.P.U.O. for year 1967-1977, source, Dept. of Employment Gazette. All the above indices were based on a value of 100.00 for 1970 in order to be comparable.

Our basic forecast for future inflation, figures needed for up to 1984, was 10% rise per year throughout. Although this is not either statistically or economically fully justified, it was an assumption good enough for the purposes, and within the limitations of this project. For comparison purposes and in order to test for robustness of the forecasts made by the methods with respect to different assumptions for future inflation other than 10% forecasts were made. The procedure will be described in due course.

The next stage was to consider all risk groups and choose the most appropriate inflation index for each one of them. This single choice was not however possible for all groups. Here we will briefly describe our choices and the reasoning behind them:

1 Liability___:__General__(Public)

The risks included in this group contain mainly compensations for faulty goods produced or sold by companies. They also contain compensations for the loss of earnings by third parties caused by the company's actions. Compensating for faulty goods could mean either or both of the following:

(a) Compensation for replacing the faulty goods.

(b) Compensation for loss of earnings, caused by the above faulty goods, to any third party. Compensations for replacement should follow the R.P.I, and Compensations for loss of earnings should follow the A.E.I.

As earnings losses seem to be the predominant element of this account, A.E.I, is to be the best choice. This opinion is shared by management.

2 Liability._j._Emp_loy_ers

This account contains risks mainly for compensations against loss of earnings caused by accidents at work. As accidents happen mainly on the shop floor rather than in offices these compensations ara for loss of earnings of manual workers. These payments ought to follow the M.W.I, which is to be our choice for this group.

Mr. D. Alexander, claims superintendent for a number of years, raised a very interesting point with respect to payments for claims of cost earnings. Following a test case, namely Lyndale Fashion Manufacturers vs Rich (1973) 1 AER 3, the amount to be paid to the claimant is to be reduced by the difference between the tax he would have paid having not lost his earnings and the tax that he actually paid. The effect on the account of this action is that as taxation increases, the average cost of each claim is to be reduced in real terms. Of course as taxation goes down the average cost per claim ought to increase. However, this effect could not be quantified during this project and therefor was not taken into account.

3 Goods_in_Transit

This account contains risks against damage or otherwise loss of goods while they are transported from one place to another. This transportation is mainly by road and rail. The account does not include marine insurance. The goods carried, mainly in bulk, include anything from scrap metals to manufactured goods. It is mainly company business. The most obvious index is WPI for this group. For wholesale prices there are several indices. These are for Input and Output of Manufacturing industries, for Commodities produced in U.K. and overseas and the like. The ideal would be to use a combination of those indices according to the mix of the business. This mix was not known and as all indices were not so different from each other we chose for the project the one of the "output of Manufacturing Industry".

The following point was made by Mr. French, the manager of the department. There is a term in the policy that limits the maximum liability to £800/ton. As inflation increases, more and more goods exceed this limit and therefore the average cost per claim, in real terms, decreases. Payments reflected this in the past but it was impossible to be quantified. However this point was noted in the analysis.

4,5,6,8 Motor:

All these accounts are very much the same in terms of the type

of risk elements they include. These are:
(a) labour (b) Spare parts prices (c) new vehicle prices (d) second-hand vehicle prices (e) garage profits (f) third party liability. No single published index includes all the above elements. A letter was written to Mr. B.D. Hudson, Statistician of the British Insurance Association (B.I. A.) asking whether his organisation produced an inflation index of motor insurance costs. The reply came saying that, although B.I. A. produces index series relating to certain costs of motor insurance, they do not produce an overall index. As we were not able to gain knowledge of the costs associated with the different risk elements of the account, we were unable ourselves to produce a single index for motor insurance. For this reason a whole *array* of indices were applied to the groups in the hope to find statistically the one that fits best. Initially, R.P.I., A.E.I., M.W.I, were applied. At a later stage and after J. Taylor suggested so, I.W.P.U.O. was appli as well, in order to reflect the cost of repair. As third party liability and Labour are very strong elements, it is thought that A.E.I, ought to apply best.

Motorj__Trade_risks

John Taylor's feeling, and in my view justified, is that this account should be removed from the motor accounts. The reason being that although it deals with cars, these are treated as merchandise rather than on the road. Risk elements are fire, theft from showroom etc. The index which best applies to this group is R.P.I, or the B.I. A. 's Index on new car prices.

9,10

These groups were altogether dropped from the analysis. They are very small and hardly any conclusions could be made.

This is a very interesting group in terms of economic analysis. Personal accident policies when sold, specify a definite amount of money to be paid, say, per week when an accident occurs and the policy holder is out of work. All payments then with respect to this policy, irrespective of when, are made according to this amount. Therefore for each cohort payments made on any development year, are in values of the year of origin rather than in values of the office year of payment. Now,policies are standardised and are not often changed. For the same premium, one gets the same compensation no matter when one buys the policy. People however, as time goes by, are prepared to pay more for insurance and therefore buy higher coverage. The question is, whether people are prepared to pay more for insurance according to what they earn or according to the general cost of living. It seems that no real answer can be given here, but the former is more likely and A.E.I, should fit best. Even in the event that inflation really applies to the group this is not according to the office year of payment, but according to the year of origin i.e. inflation does not apply diagonally in the runoff triangle, but vertically.

12,13 _Prop_erty_

The "new lamps for old" policy was introduced in 1969. This means that anything insured and lost will be paid by the insurer for the price of new. This is clearly related to R.P.I, and indeed the company's opinion is so.

14 Y§cht_and_motor_boat

First introduced in 1972 and thus is not old enought for definite conclusions to be made. Includes not sea-going vessels. The risk elements are similar to the motor accounts and in particular to group 6. Management's opinion is that theft is the predominant element. Still, no definite decision about which index is more appropriate can be made. The reasons are the same as for the motor accounts.

4.3 Test Application - Initial Analysis

Having decided which inflation indices are most appropriate for each risk group and having programmed the methods in FORTRAN IV we were now ready to apply the methods and analyse results for only one group initially. The results of this study we would discuss with management and after accommodating their remarks and suggestions, we would proceed to analyse the behaviour of the methods for other risk groups.

The group chosen to be the "test group" was No.4 (motor - private cars). The choice was dictated by a number of reasons. First, group 4 is the largest account of the company and thus it had to be analysed anyway. Secondly group 4 is a fairly stable account with no dramatic changes in terms of business mix apart from a gradual trend towards third party rather than comprehensive insurance. It also seemed to be the most familiar account to John Taylor and therefore we would be able to obtain valuable comments from him when discussing the results. We were further interested to see whether we would establish which index of inflation is most appropriate, statistically since we were unable to do so on economic grounds.

We have already discussed the grounds on which we will test the methods and we will just recall them here:

(1) Fit of model assumed for the runoff triangle by each method, to actual runoff triangle.

(2) Reliability of forecasts of future payments given by each method.

(3) Robustness with respect to different forecasts for future inflation.

(4) Reaction of forecasts given by each method to additional observations.

For the above tests three inflation indices were used, R.P.I., A.E.I., and M.W.I. All tests apart from 3, were carried out using all three indices. We hoped, before seeing any results at all, that a statistical way to establish which index is most appropriate for the group would be to observe the correlation coefficient between the X's derived by the separation method and the index. This was accommodated in our program.

To test for (1) we simply had to model the runoff triangle using the appropriate formulae for each method of Appendix A.

To test for 2 we decided to apply all methods to a runoff triangle including years of origin up to 1975. Then after obtaining forecasts of payments for 1976-1977, to compare these with the actual observations. Doing so we assumed that being in 1975 we could have exact knowledge of the inflatioi for 1976 and 1977. This would enable us to test the forecasting value of the method without introducing additional errors by assuming inflation figures different from the actual.

To test for 3 we decided to use only the R..P.I. The idea was that we were concerned only for different forecasts and not for the choice of inflatioi index as such. Therefore whatever results we would get using R.P.I, we would also get by using any other index. We would perform the test by using each method for forecasting future payments under one assumption for future inflation and compare results with ones under a different assumption. Results for each set of assumptions were compared against results for our basic 10% p.a. forecast.

To test for (4) we applied each method to: (a) runoff triangle including years of origin up to 1975; (b) runoff triangle including year of origin up to 1977. Keeping assumptions for future inflation constant we compared forecasts, made using (a) to ones made using (b).

As statistical measures of'fit" the following were chosen:

(i) Sum of squared differences viz. Σ (modelled - actual)^2 or where forecasts were compared with forecasts:

$$\Sigma \text{ (forecast 1 - forecast 2)}^2$$

(ii) A statistic similar to chi-squared:

viz. $\dfrac{(\text{modelled-actual})^2}{|\text{actual}}$

The first would provide a measure of the deviations in "straight pounds" whereas the second would give a measure of relative deviations. The second statistic gives a better measure statistically but the first would be better on financial grounds where all pounds are the same. In fact both statistics were used only for tests 1 and 2. For 3 and 4 only the sum of squared differences was used. The idea was that since we did not have any actual observations we could not use the second statistic.

4.4 Discussing the Results with J. Taylor - Comments - Alteration of Analysis Line - Established Line

The results of the above analysis were arranged into Exhibits and discusse with John Taylor during a meeting that took place in Kendal. During this discussion the following comments were made:

Inflation Indices; John Taylor suggested we use an index reflecting payment rates rather than wages in addition to the three indices we used so far. Such an index ought to reflect labour costs of the motor account. We accepted the suggestion and agreed to incorporate the index into our analysis. This is I.W.P.

<u>Test 2 : Reliability of forecasts:</u> To test this we applied the methods to data up to 1975 and then compared forecasts for 1976-1977 to actual observations. The problem was that for group 4 these forecasts were higher than the actual payments, irrespective of the method they were obtained by. We therefore agreed to, in addition to this, apply the methods to data up to 1976 and compare forecasts for 1977 to the actual. It was agreed that this procedure was the only recommended one for testing the reliability of forecasts.

<u>Test 3 ; Robustness with respect to different forecasts for future inflation:</u>
There are different merits for a method that is robust and a method that is not so. The chain ladder method is insensitive to inflation and therefore is not favoured by actuaries. On the other hand to have a very sensitive method means that a lot depends on one's ability to accurately forecast inflation. If one is usually accurate in these forecasts then a sensitive method is appropriate. If not, then a robust method is appropriate.

The analysis on this test was decided to be kept for group 4, but not to be carried out for any other group. This was because the results given for group 4 coincide with results given for other groups.

<u>Test 4 ; Fraction of forecasts to additional information</u>
Although this test was designed to compare forecasts against forecasts, this is now investigated implicitly by test 2 as modified, to compare forecasts, given in different periods to the actual. Apart from this explicit comparison an implicit one is made between forecasts. Therefore this line of analysis is to be abandoned for any other group.

<u>Statistical measures of "goodness of fit"</u>
It was agreed that the company was not only interested for "straight pounds" but also for relative deviations. The sum <u>of squared deviations</u> (s.s.) was replaced by the Mean Square Error (M.S.E. $= \sqrt{\frac{s.s.}{\text{No. of observations}}}$) in order to produce figures comparable to the observations. The χ^2 statistic was replaced by a similar statistic which would reflect better the relative deviations. This was named $x^2 = \Sigma\left(\frac{\text{Actual-modelled}}{\text{Actual}}\right)^2$. Both statistics were to be used for all tests.

Cost per claim method.

To produce A's needed in the cost per claim method (see Appendix A2) we had used data prior to 1970. No such data was possible to be used by any other method. For the sake of comparison of methods we ought to use exactly the same data for all methods. So we decided to do , although the ability to use additional data was recognised as a merit of the cost per claim method.

The cost per claim method in this initial analysis performed poorly compared with other methods. We decided to especially analyze the cost per claim method using three different ways to obtain the A's (see Appendix A2).

(a) As a simple average

(b) As. a weighted average (1970=1, 1971=2 etc.)

(c) As a subjective estimate for each development year

This analysis was only carried out for group 4 and using only R.P.I.

Presentation of results :

For each group we would carry out the analysis we would produce the following set of results:

The first page of the set would contain the data used for the analysis (see Appendix C).

Exhibit A (see Appendix Dl) This would contain the actual runoff triangle and all models of it as assumed by each method and each index. Also measures of fit will be included.

Exhibit B; would contain the forecasts, made by each method and each index, at the end of 1977.

Exhibit C: (see Appendix D2) would contain forecasts made in 75 as compared with actual, and forecasts made in 76 as compared with actual. It will also contain measures of how well these forecasts fit to the actual observations.

Exhibit D; (see Appendix D3 only for group 4) This would contain information about robustness of the methods to different forecasts of inflation. The forecasts tested against our basic 10% p.a. are (a) forecasts made by a Box and Jenkins mixed model with p=2 q=2 given R.P.I, for up to 1977. (b) 15% p. a. (c) 10% for 1978, 1979 and then 15% p.a. Includes measure of goodness of fit.

Exhibit E: (see Appendix D4 only for group 4): This would contain comparisons of forecasts made at the end of 1975 against ones made at the end of 1977. Again the measures of goodness of fit are included.

5. Applying the established line of analysis to data of various risk groups - Results

5.1 To Which groups the analysis was applied

The modified line of analysis, as described in the previous chapter, we decided to apply to data (see Appendix C) from the following groups, apart from group 4: 1 and 2 : Liability 3 : Goods in Transit 5 : Motors :Commercial vehicles

11 : Personal Accident

12 : Property : Theft.

My responsibility was groups 3, *5* and 12.

Group 3 was chosen mainly because we were interested to see how much, the performance of the different methods, was affected by the £800 per ton limit on payments which was discussed earlier.

Group 5 was chosen primarily in order to verify results obtained from group 4, as it is - very similar to this group, although the size of the account is much smaller. However it contains the same risk elements.

Group 12 was chosen because it was a "clear cut" group in terms of inflation index. We were interested to see whether this was statistically justified.

5.2 Applying Methods to Data from Group 4.

Test 1 ; Fit of Models Assumed for the Runoff Triangle by early method, to actual Runoff Triangle (For results see Appendix Dl.l)

The lowest M.S.E. figure was produced by the fit of the separation method's triangle to the actual, followed by the M.S.E. of the chain ladder method and the chain ladder method with inflation. Indeed these two were not significantly different apart from when the M.W.I, was used, in which case the M.S.E. of the chain ladder with inflation was a high as the one produced by the cost per claim method.

The x2 produced by the chain ladder method with inflation was the best (lowest), followed by the x2 of the chain ladder method with no significant difference. The cost per claim method did better than the separation method which gave the highest x2.

Test 2 : Reliability of Forecasts

(For results see Appendix D1.2) Forecasts made in 1975 as compared with actual

Forecasts by all methods and indices are higher than the actual observations they are compared with. The chain ladder method produces figures closer to the actual apart from when M.W.I, is used in which case the separation method does so and when I.W.P.U.O. is used in which case the chain ladder method with inflation does so. In both cases the difference is not significant.

In terms of M.S.E the separation method produces the lowest figures when RPI and MWI are used. When AEI and I.W.P.U.O. are used the chain ladder method with inflation produces the lowest figures.

The chain ladder method produces the lowest x2 figures. For the second and third places the separation method and chain ladder with inflation method behave as they did with M.S.E.

Forecasts made in 1976 as compared with actual

The chain ladder method produced figures very close indeed to the actual in terms of their sum. In terms of M.S.E. again the chain ladder method gave the lowest figure apart from when M.W.I, was used where the separation method did so. In this case the chain ladder method, gave the second best. The x2 figures given by all three chain ladder, chain ladder with inflation and separation methods were very much the same apart from when the A.E.I, was used, in which case the separation method gave a x2 as high as the one by the cost per claim· method.

Test 3 ; Robustness with respect to different forecasts for future inflation

(For results see Appendix D1.3)

The chain ladder method is completely insensitive with respect to such changes and indeed this is its main weakness as seen by actuaries. As far as the three other methods are concerned the chain ladder method with inflation gave the lowest M.S.E. figure, whereas the M.S.E. figures given by the other methods were very much the same. The chain ladder method gave, however, the highest x2 figure and the other two methods produced x2 figures which are indistinguishable. As x2 here are less reliable than M.S.E., because the denominator itself is a forecast, we can say that on the grounds of M.S.E. figures observed, the chain ladder with inflation method is more robust without this to imply that the other two methods are over-reactive.

Test 4 : Reaction of forecasts to additional information

(for results see Appendix D1.4)

The M.S.E. figure produced by the chain ladder method with inflation is best (highest) when R.P.I.'s used followed by the chain ladder method. When A.E.I, is used the separation gives the highest M.S.E. followed by the chain ladder method with no significant difference. When M.W.I, and I.W.P.U.O. are used the chain ladder method gives the highest figures. The x2 produced by the separation were highest followed by the chain ladder method's apart from when the I.W.P.U.O. was used in which case the cost per claim method gave equally high figure. The x2 are again less reliable than M.S.E. On the grounds of M.S.E. the chain ladder method is overall more reactive to additional information.

5.3 Applying Methods to Data from Group 5

For this group the same inflation indices were used as for group 4.

Test 1 : (For results see Appendix D2.1)

In terms of M.S.E. again the separation method gave the lowest (best) figures in all cases. It is followed by the chain ladder method with inflation.

The x2 produced by the separation method is again the highest. The best figures are produced by the chain ladder method followed by the cost per claim method.

Test 2 : (For results see Appendix D2.2)

Forecasts made in 1975 as compared with actual

Once again forecasts are higher than actual figures. The closest to the actual were given by the separation method when R.P.I., M.W.I., I.W.P.O. were used and when A.E.I, was used were given by the chain ladder method.

In terms of M.S.E. the separation method gives the lowest figure for R.P.I., M.W.I., I.W.P.U.O. and the chain ladder with inflation when A.E.I, was used.

The x2 produced by the separation method were lowest when R.P.I, and M.W.I, were used whereas the chain ladder method produced the lowest figures for A.E.I, and I.W.P.U.O.

Forecasts made in 1976 as compared with actual

The forecasts given by the chain ladder and separation methods for R.P.I, and I.W.P.U.O. were equally close to the actual. For A.E.I, the chain ladder with inflation method gave the closest results and for M.W.I, the chain ladder.

The separation gave the lower M.S.E. for R.P.I, but the chain ladder with inflation did so when A.E.I, was used. For M.W.I, and I.W.P.U.O., the separation and chain ladder with inflation gave equally good results.

Conclusions for Groups 4 and 5

The chain ladder method performed well for group 4 although this was not the case for group 5. On the other hand the chain ladder method with inflation and the separation methods performed almost equally well for both groups. For group 4 the chain ladder method with inflation was slightly in the lead whereas the separation method was marginally better for group 5. The above conclusions are based on the results of the tests 1 and 2. Therefore it is recommended that, for the purposes of forecasting, the chain ladder with inflation ought to be used together with the separation method for these two groups.

No clear judgement was made on statistical grounds about which index fits best. The correlation coefficients proved unreliable measures and hence we chose as our measure the fit of the hypothetical triangles to the actual. Again in this case no clear cut decision could be made.

5.3 Applying the Methods to Group 3

On economic grounds the W.P.I. ought to fit best to this group. For the purposes of comparison and in order to demonstrate this I carried out the analysis using both W.P.I, and R.P.I. The astonishment came when the R.P.I, gave better statistical measures of fit for both tests 1 and 2. This can be explained though by the fact that we used as our W.P.I, one of the number of wholesale prices indices without considering the mix of business. A further explanation could be the effect that the £800/ton limit in the contract has on the account. It might be work a try to establish the business mix and to quantify the effect of this term in the contract.

Test 1 : (For results see Appendix D3.1)

In terms of M. S.E. the chain ladder method gives the overall best result followed very closely by the chain ladder with inflation.

In terms of x2 the separation method gives by far the best result.

Test 2 ; (For results see Appendix D3.2)

Forecasts made in 1975 as compared with actual

In terms of forecasts the chain ladder gives the closest to the actual with the chain ladder with inflation very close in the second place. In terms of M.S.E. and x2 the separation method produces marginally better figures than the chain ladder with inflation method.

Forecasts made in 1976 as compared with actual

For the R.P.I, the separation method gives the closest to the actual forecasts. For the W.P.I, the chain ladder with inflation does so. In terms of M. S.E. again for R.P.I, the separation produces lowest figures and for W.P.I, the chain ladder with inflation. The x2 of the separation are slightly better than the ones by the chain ladder with inflation.

Conclusions

The separation method and the chain ladder method performed very similarly for this group. The separation method performed slightly better. In order to forecast it would not be a bad idea if both methods were applied for comparison purposes.

5.4 Applying the Methods to Data from Group 12

Here again although R.P.I, ought to apply best to applied W.P.I, as well, for the sake of comparison. The results of test 1 were better for R.P.I, a fact that confirms our economic consideration.

Test 1 : (For results see Appendix D4.1)

In terms of M. S.E. the chain ladder with inflation performed better followed by the chain ladder method.

The chain ladder method was again second best for x2 but this time the separation method gave overall best figure. Test 2 : (For results see Appendix D4.2)

For both comparisons i.e. 75 forecast against actual and 76 forecasts against actual, the totals produced by the chain ladder method were the closes to the actual,

The statistical measures produced by the two methods i.e. M.S.E. and x2 for both comparisons were very much in line with the chain ladder method slightly in the lead.

Conclusions

For the purposes of forecasting the chain ladder method performed better for this group. If a method taking account of inflation ought to beused this is to be the chain ladder method with inflation.

6. General Conclusions ~ Comments - Limitations

The project did not deal with the problem of estimating provision for year of development after k+1. It also did not deal with actual distribution All our analysis was concentrated with averages. Further research must be carried out with respect to the above two points in order that the value of the methods be fully investigated.

A considerable amount of economic research ought to be carried out in order to establish which inflation index fits to each group. The mix of business ought to be quantified for many groups and primarily for the motor ones. Effects on accounts caused by elements such as the £800 per ton limit in group 3 ought to be quantified.

Despite the above limitations the project has carried out a valuable analysis for Provincial and must be regarded as a first step towards a full investigation of the relative merits of the methods.

The package in FORTRAN IV that resulted from our work must be regarded by Provincial as a valuable asset.

Appendix A : Mathematical formulae for modelling the Runoff Triangle and. Estimating provisions for outstanding Claims

Appendix Al

Chain ladder method

Modelling the runoff triangle

$C_{..}$ the observed payment in cohort i and year of development j.

The expected payment is:

$$E_{ij} = \sum_{k=Q}^{7-i} C_{ik} \;\; \chi\, r = l \text{-.----} \quad /\text{-}i \;\; k=0 \;\; k$$

for j = 0.. . (7-i), where

$$f_j = (f_j - 1) / \prod_{k=j} f_J \qquad j \geq 1, \quad r = 1/\prod_{k=0} f \quad \text{where}$$

$$f_k = \sum_{n=0}^{k} \; . \; \sum_{i=0}^{7-k} O_{\Lambda} \; m \cdot / \; \cdot \; / \sum_{i=0}^{/7-k} \; \text{-.} \; « \sum_{m=0}^{k-1} C_{im} \; \mho \; f = 1$$

Estimating provisions

Define $A_{..} = ,\sum_{h=0} C_{..ih}$, $m_j = \sum_{i=0} A_{i.j+} \; . \; \sum_{i=Q}^{k-j-i} A_{ij}$, j = 0 ... k-1

m_j is taken as an estimate of $\sum_{n=0} \mu_{lh} / \; \mu_i$ $\sum \;\; \text{ih} / \;\; / \sum_{n= Jh=0}^{J} y.,$

Let $m_k +$ be an estimate of $\sum_{n=0} \mu_{in} \; / \; \sum_{n=0} \mu_{in}$ (In our project this estimate was taken as 1). Define

M = m.m. ...m +, j=0...k. The provision for claims is then

$P_i = A_{i,k-i} \,.\, CM_{Tc-i} \; .\text{-}1) \quad i=0 \; ..... \; k$

Chain Ladder method with Inflation

Modelling the runoff triangle

$^{r}7\text{-}i \; /C.. \qquad _{r}. \; E.. = \iota \; \Sigma$

-iJL· U -- χ I. .

/k 7-k C. \ //7-k k-1 C.

f - | τ T im /ν Σ —ui I , f

k I Vm=0 i=0 i+m// \i=0 I m=0 // I i"' / 0=1

where I_{i+m} is the inflation figure for office year i+m.

Estimating Provisions

Let m_K be the real value version of HL (again in our project m_{K} assumed 1), then

1 I, I I

I I

where the numerator follows the convention

that it is m-1 when j=0 and equals 1 where j = -1. Then

P - A

J. A, , .

ι ι,k-1

7^k \ ι
k-i Σ(S.I. . J-0 J " J

Appendix A2

Cost per claim method

Modelling the runoff triangle

Let A. =| Σ k-j-1 C.. γ3— i+j (k~j-l)be the real value hypothetical cost per claim for development year j. The runoff triangle for the number of claims is modelled by the chain ladder method. After each cell is multiplied by the appropriate A., we reflate back again to actual values.

Estimating Provisions

A. are then again applied and reflation to actual values is done using the

Provisions for number of claims are made using the chain ladder method. The A. are then again applied and reflation to ;

appropriate forecasts for future inflation.

Appendix A3

Separation Method

Modelling the Runoff Triangle

$$E_{..} = \lambda_{..} \chi\, r. \quad \Sigma_{J}\, r_{.} \, \chi^{7-i}_{k=0} \; {}^{\Lambda}_{k} \; \Sigma_{n=0} \; \frac{7\, in}{}$$, where X's and r's are

estimated as follows.

Estimating provisions

Let n_{ι}. be the total number of claims in cohort i. Then

$$S_{ij} = C_{ij}/n_{\iota} \;, \quad V_{j} = \sum^{k-j}_{=} S_{ij} \;, \quad D_{\eta} = \sum^{\eta}_{\iota=0} \, {}^{Z}A_{-i,i'}\, {}^{r}j \quad {}^{P}j \quad {}^{7} \quad \eta=0$$

Then r. and λ are respectively estimated by

$$r_{J} = V_{.} \; / \sum^{k}_{n=j} \lambda \quad j=0 \,.....\, k \text{ and } \lambda = D \left(1 - \sum^{k}_{j=h+1} r_{J}\right) , \; n=0,...,k$$

$$\text{The provision } P_{\iota} = A_{i,k-i} \; \frac{\left(\sum^{k-i\wedge} \Gamma.\lambda^{*}. \;.+ \sum^{k} r.I \;.\right)m.T}{\sum_{j=0} r_{J} X.} - 1$$

$m_{ix} + = 1$ again.

Appendix B ; Returns Format for Reported Claims

Explanation of Headings

The attached page is an example of how returns for reported claims were formulated. We will explain here the headings for each column.

Year of origin: The year that the accident that caused the claim happened.

Settled this yea.r;

nil; Number of claims settled during the reporting year without cost to the company.

Other; Number of claims settled during the reporting year at some cost to the company (recoveries are negative costs).

Claims 0/S at year end: The definition of oustanding is rather different from the one generally accepted throughout the project. Here outstanding means claims notified but not settled. The general definition of the project includes all claims to be settled, irrespective of whether they are notified.

Total claims: The sum of settled and 0/S claims.

Total payments this year: Payments made with respect to the above claims during this year.

Total paid to date; Payments made with respect to the above claims, not only during this year, but also in previous years.

Claims 0/S paid to date; Payments made with respect to outstanding claims during this or previous years.

At end of year estimate: Case estimates for the outstanding claims.

Total paid to date plus estimate: The sum of the amounts in the seventh and ninth columns.

At the bottom of the page there is an analysis .of the claims notified during this year. It is clear that the total of the fifth column includes claims notified in previous years.

ACCOUNTS DEPARTMENT -GPEHKO

RISK GROUP 04

RDVFK i'fSw CLAIMS RETURNS e 'J. K.
FULL QLAIKS ANALYSIS - 1977

WM N^M^f: ^CL^53 τΓiiN Π1ΛTE 15/03/78
PAG2 NO. 5

;TTLED THIS YEAH NIL
YEAR OF •- OTHER
ORIGIN

YEAR OF ORIGIN	SETTLED THIS YEAR NIL	OTHER
	O	O
1901	0	0
1902	η ι 0 - --	
1907		0
I960	0 •--.	0
1963	0	0 0
1965	2	
1966	0 ---	η
1967	0	1
1968	- 0	__, g
1969	1 1 4 3 33	---ᒐ
197Q		29 47
1972	3891 5964	88 237
1973		854
1974	9089	8249
1975		-16164
1976		25688
1977		
TOTAL		

IMS 0/3 EAR END	TOTAL CLAIMS----		TOTAL PAYMENTS THIS YEAR -	TOTAL PAID TO DATE	CLAIMS O.S, PAID TO DATE	AT END OΓ FSTIHATE		TOTAL PAID TOYGAR DATE * ESTIHATi
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17	29 ._.	38,	931,26	3,453,963,97	65,526,02	35, 720. no	3,	489.n83.97
12	42	64,	959,66	4.098,305,12	49,2₀B,33	113, 840, no	4,	212, 145.12
3p	81	160,	245, в!	4.416.37^,63	154,726,47	271, 700, no	4,	680, Q75.63
• 7 0	16.6		451,03	4,668,24g,93	102,159,50	308,010. no	4,	976,255,93
185. 1Q8	373	299,	165,95	4,637,610,43	237,455.90	340,655,00	4,	978,665,43
r:- 336	1377- .--.-,	354,	342',07	4,836,487,11	241,089,21	I,Γ.74,095,00	5,	910,582.11
114Z	1220.7	1,620,	990,07	4- 702-749, 06	457,350.44 -	2,163.660,00	6,	866,409, Q6
11189	33317	3,954,	497',33	3.954,497,33 -	1,204,282,68	5,0^Q7,I55,no	9,	051,652,33
12^21	476?8 -	6,674,	649,97	37,819,19£,B1	¡>,5₂n,966i3?	9,411,755.00	47,	230.950, 51

riFiep THIS YEAR 3001 659.950,67

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Appendix B: Example of Returns Format for Reported Claims to»

Appendix Cl

DATA FOR GROUP 3

Payments

Year of Development

	1	2	3	4					Case Estimates O/stg.	IBNR	TOTAL
70	273981	241941	23193	13005	453	385	829	0	-	-	-
71	295682	218835	44463	1325·;	4134	522	204		60	-	60
72	425147	286322	26278	8358	3273	-4342			2030	-	2030
73	407671	358877	47125	9609	10512				38600	287	38887
74	568064	416765	52866	20356					32105	693	32798
75	601597	488009	52219						104475	7317	111792
76	785404	642848							425640	14578	440218
77	927000								1533393	359100	1892493

No. of Open Claims

Year of Development

	1	2	3	4	5				Estimate of Cohort Size (1st year rep + IBNR)
70	5670	2781	316	.54	13	. 4	2	0	6820
71	5423	2804	288	67	16	10	5		6523
72	5977	3015	335	74	32	11			7297
73	5727	3207	411	94	37				7203
74	6304	3354	463	119					7891
75	6642	3663	483						8236
76	7424	4521							9256
77	8064								10116

Appendix C2

DATA FOR GROUP 4

Payments

Year of Development | Case Estimates

									Out standing	IBNR	Total
70	1792437	945460	329336	167348	77777	73147	31276	38931	35720	-	35720
71	2119796	987912	330688	268108	228681	97934	64959		113840	-	113840
72	2179185	1197884	455125	261011	170122	160245			271700	276	271976
73	2497770	1327813	479319	177891	185451				308010	3910	311920
74	2555319	1369292	414033	299165					340855	13050	353905
75	3098436	1383708	354342						1074095	22972	1097067
76	3081758	1620990							2163660	22893	2186553
77	3954497								5097155	832500	5929655

No. of Open Claims | Estimates of Cohort Size (1st year reported + IBNR)

Year of Development

									Cohort Size
70	35163	12561	1288	399	152	74	40	29	37913
71	36130	12527	1226	386	165	79	42		39130
72	36619	13251	1347	443	183	81			39719
73	35987	12952	1368	389	166				39187
74	31504	11893	1213	378					34732
75	31605	11808	1377						34447
76	28681	12287							31253
77	33317								36647

Appendix C3

DATA FOR GROUP 5

Payments

Year of Development

2 3 4

70	245183	157324	43538	29310	16855	-185 256 , -12
71	254471	153276	84025	57939	15185	4828 £439
72	320876	181382	38144	43054	0	8980 i
73	323013	250380	105426	40115	36199	
74	334673	214054	64261	45651		
75	450014	254756	52089			
76	506942	321960				
77	732912					

Case Estimates 7

0/stdg.	IBNR	TOTAL
10	0	10
61005	0	61005
6605	0	6605
26280	0	26280
168000	1937	169937
128070	313	128383
303845	3665	307510
983005	160950	1143955

No. of Open Claims

Year of Development

Estimates of Cohort Size

(1st year rep. + IBNR)

									Estimates of Cohort Size
70	6390	2637	295	66	15	7	4	1	7140
71	5567	2422	248	60	22	13	11		.6267.
72	6176	2648	241	56	0	13			6976
73	5930	2703	265	62	24				6630
74	4964	2368	268	65					5664
75	5638	2541	282						6342
76	5524	2895 !							6252
77	6764								7634

Appendix 04

DATA FOR GROUP 12

Payments

	Year of Development 1	2	3						Out-Standg.	Case Estimates IBNR	TOTAL
70 71	303622	121552	8650	1556	890	357	53	123	50	-	50
72 73	378762	144073	20818	3879	-1982	340	234		5350	-	5350
74 75	462912	189314	4865	7187	826	63			3500	789	4289
76 77	447531	196377	12281	7916	1256				160	4682	4842
	894267	442200-	30000	12821					23360	12459	35819
	1348734	568132	44726						37510	37510	70039
	2094425	851090							289250	35733	324983
	2927984								1801209	594990	2396199

No. of Open Claims

	Year of Development 1	2	3						Estimates of Cohort Size (1st year rep. + IBNR)
70 71	7762	1992	122	29	10	4	5	3	8612
72 73	8140	2174	123	32	19	12	7		9015
74 75	9315	2489	143	38	12	12			10522
76 77	9685	3422	204	43	22				11168
	18605	5320	299	64					20893
	22907	6166	413						26060
	31975	8339							36147
	37887								43296

Afpendix D1.1 : Results of test 1 for group 4

	M.S.E.	
Chain Ladder	62024.9	1.348
Chain Ladder Inflation (R.P.I.	67115.5	1.271
(A.E.I.	65301.5	1.278
(M.W.I.	118632	1.331
(I.W.P.U.O.	62611.8	1.180
Cost/Claim Inflation (R.P.I.	105723	1.579
(A.E.I.	136581	1.639
(M.W.I.	120507	1.668
(I.W.P.U.O.	112115	1.734
Separation	43881.4	2.978

Appendix D1.2 : Results of test 2 for group 4

	M.S.E. 75 vs 77	M.S.E. 76 vs 77	X2 75 vs 77	X2 76 vs 77
Chain Ladder	149252	81959	1.04372	.50796
Chain Ladder Inflation (RPI	146201	85928	1.61928	.529114
(AEI	119751	84758	1.27302	.463898
(MWI	122741	101605	1.24335	.482198
(IWPUO	99349.5	81599	1.16827	.430969
Cost Per Claim (RPI	143474	103489	1.57324	.652829
(AEI	162701	96902	1.69013	.645187
(MWI	194200	103300	1.95011	.689481
(IWPUO	144776	141226	1.4767	.628008
Separation (RPI	126376	94588	1.32331	.493572
(AEI	202176	145420	2.27491	.655475
(MWI	112425	72259	1.12522	.432861
(IWPUO	151344	90972	1.55162	.484142

Appendix D1.3 : Results of test 3 for group 4

	10% Forecast against 10% for 78, 79 and then 15%		10% Forecast against 15% p.a. Forecast		10% Forecast against P=2, Q=2 model	
	M.S.E. 0	X 0	M.S.E. 0	X 0	M.S.E. 0	X 0
Chain Ladder Cost/Claim	10594	0.243641	32170	0.82221	39893	.45999
Chain Ladder + Inflation	9868	0.306314	29734	0.869931	36360	.4955
Separation	11174	0,243636	33182.9	0.825208	29760	.46106

Appendix D1.4 Results of test 4 for ^roup 4

		M.S.E.	X2
adder		33453	0.230302
Chain Ladder, Inflation Cost/Claim	(RPI	36154.2	0.138731
	(AEI	28332	0.199588
	(MWI	25959	'O. 205861
	(IWPUO	26198	0.231661
	(RPI	25715	0.18111
	(AEI	28767.9	0.21814
	('MWI	26039	0.214325
	(IWPUO	26255	0.311894
Separation	(RPI	32456	0.30999
	(AEI	35254	0.310004
	(MWI	29615	0.310018
	(IWPUO	32208	0.310006

Appendix D2.1 ; Results of test 1 for group 5

Chain Ladder

Chain
Ladder + (

Cost/Claim(
(

Separation

	M.S.E.	X^2
:r	17044	2.8033 E+8
[RPI	17206	3.19087E+8
:AEI	16503	3.24217E+8
;MWI	16211	3.2877 E+8
: IWPUO	14901	3.18409E+8
;RPI	17808	2.8234 E+8
:AEI	20566	3.05236E+8
;MWI	25267	3.19909E+8
: IWPUO	19792	2.8737E +S
	14909	3.27357E+8

Appendix D2.2 : Results of test 2 for group 5

(AEI Separation (
(MWI

(IWPUO

Chain Ladder

(RPI

Chain Ladder(AEI +

(MWI Inflation

(IWPUO (RPI (AEI

Cost

Per

Claim

(MWI (IWPUO (RPI

M.S.E.		X^2	
75 vs 77	76 vs 77	75 vs 77	76 vs 77
24952	20828	3.325E+8	2.540
29162	21954	4.55 E+8	2.58
24297	19683	4.33 E+8	2.33
24372	21789	4.39 E+8	2.29
21116	19131	4.21 E+8	2.27
26837	24754	5.34 E+8	3.10
29360	23603	5.69 E+8	2.99
33937	24133	5.9Γ E+8	3.06
26293	23280	5. HE +8	2.91
18882	19193	3.14 E+8	2.34
27341	22157	4.1 E+8	2.51
16510	22705	3.08 E+8	2.25
20653	19243	3.52 E+8	2.33 j

Appendix D3.1 : Results of test 1 for group 3

	M.S.E.	
Chain Ladder	11886	86.863
Chain Lad(RPI Inflation(f Wlr J-	13217	56.030
	12129	49.135
Cost/ Claim	36746	48.538
(RPI	45585	48.642
((WPI	46774	10.168
Separation		

Appendix D3.2 : Results of test 2 for group 3

	M.S.E.		X^2	
	75 vs 77	76 vs 77	75 vs 77	76 vs 77
Chain Ladder	8874	12813	2.151	12.292
Chain (RPI	6894	9497	2.689	14.075
Ladder +(Inflatio4WPI	7643	6035	2.780	15.325
Cost/ ,	19396	21390	2.861	15.750
Claim (WPI	24439	35932	3.086	16.886
Separ- \	5661	6234	2.192	13.392
ation (WPI	6294	14703	2.256	14.999

Appendix D4.1 : Results of test 1 for group 12

	M.S.E.	X^2
Chain Ladder	12967	29.815
Chain Lad. + Infl. (RPI	8854	31.598
(WPI	11504	33.495
Cost/claim (RPI	196076	76.172
(WPI	223744	82.507
Separation	13175	19.065

Appendix D4.2 ; Results of test 2 for group 12

	M.S.E.		X^2	
	75 vs 77	76 vs 77	75 vs 77	76 vs 77
Chain Ladder	8583	23001	64.424	43.853
Chain Lad. + Infl. (RPI	8679	22173	85.175	56.337
(WPI	7469	31919	94.609	62.601
Cost/Claim (RPI	52948	109401	2.925	98.247
(WPI	60115	135222	79.051	104.015
Separation (RPI	68295	162127	130.993	100.785
(WPI	73882	188143	145.528	112.316

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